Proceedings of the 11^th Brazilian Congress of Thermal Sciences and Engineering – ENCIT 2006
Braz. Soc. of Mechanical Sciences and Engineering – ABCM, Curitiba, Brazil, Dec. 5-8, 2006

Paper CIT06-0478

THE REYNOLDS STRESS MODEL FOR PREDICTING THE WIND FLOW WITHIN AND ABOVE A FOREST CANOPY

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Abstract. *The turbulence structure of the two-dimensional (2-D) flow within and above a reduced model of a forest was investigated with the turbulence Reynolds Stress Model (RSM). The results were compared with numerical simulations performed with the Standard $k-\varepsilon$ Model (SKE) and also with experimental data from wind tunnel experiments. The model forest has a uniform height of 15 cm and a length of 6 m. The arrangement for the distribution of the trees studied was 125, corresponding to leaf-area indexes of 1.7. The RSM provided a good prediction of mean wind speed and Reynolds stress, but overestimated the turbulence intensity. One important result from the study indicates that there is no clear advantage in using the RSM versus the SKE in the simulation of the canopy flow.*

keywords: *Canopy flow, Reynolds stress model, $k-\varepsilon$ model, turbulence, wind tunnel experiments.*

1. Introduction

The subject of the flow within and above plant canopies has attracted considerable research attention in the past 20 years. The motivation for these studies can be found in a number of practical applications. Examples include wind load estimates, hydrology, meteorology and soil science. In addition, the studies of the turbulent flow in the plant environment are crucial to understanding of the processes that governing mass, momentum and heat between the vegetation-atmosphere systems.

Measurements from field and wind tunnel experiments (Gardiner, 1993; Shaw *et al.*, 1995;) had provided insight into the turbulence structure and the process that govern the mean air flow and turbulent field. Recent works include the results from the University of Bristish Columbia Mechanical Engineering wind tunnel experiments together with comparisons from field experiments (Novak *et al.*, 2000; Novak *et al.*, 2001) and numerical models (Phaneuf *et al.*, 2004). Other recent study in neutral condition was performed out by Katul *et al.* (2004), which investigated different classes of the $k-\varepsilon$ models (and simplifications to them) and compared the predictions against data sets collected in eight vegetation types and in a flume experiment.

In this context, the numerical models for predicting the canopy flow have been developed. However, accurate prediction of the wind and turbulent flow are difficult due to the complexity in the array of plant elements (branches, leaves, trunk) and complex dynamics of air momentum transport within a canopy. In order to describe the behavior of the canopy turbulence it is necessary to compute, at a minimum, mean flow, turbulent kinetic energy (TKE), some partitioning of TKE among its three components, and Reynolds stresses (Katul, 2001b).

The first model studies of the canopy flow were based on the K-theory or gradient-diffusion theory (Cowan, 1968; Thom, 1971). Such models may well reproduce the mean velocity, but cannot provide second-order statistics. The limitations of the first-order closure were described by several authors. According Wilson and Shall (1977) these models provide little insight into the nature of the momentum transport processes within the vegetation canopy. Pereira and Shaw, (1980) pointed out that first-order closure modes can not provide accurate predictions of the wind velocity in the lower portion of a plant canopy, where a near-zero vertical gradient of the mean wind velocity is frequently observed (Shaw, 1977). In consequence, a second-order closure approach, such as the Reynolds stress model (RMS), has been proposed to simulate the turbulent transport in forest canopies. In this approach, the conservation of the streamwise mean momentum coupled with budget equations for tangential stresses results in three terms that require closure modelling, namely, the pressure-strain term, the momentum flux-transport term, and the turbulent kinetic energy dissipation term (Katul and Chang, 1999). This model provides a practical framework for computing the velocity statistics for modeling scalar transport within the canopy sub-layer. Katul and Albertson, (1998) reported that the deficiency of the second-order closure models is the flux-gradient approximation of the triple-velocity correlation, because produces unrealistic momentum flux transport profiles near the canopy top . Meyer and Paw U, (1986) in order to avoid the deficiency in such models developed a third-order closure model for the triple-velocity correlation, which related the fourth moment to second moments using a zero-fourth cumulant expansion. This model was able to predicted wind profiles satisfactorily within a number of very different canopies. However, the model that was proposed by Meyer and Paw U, (1986) includes 10 equations for only a one-dimensional problem, and are computationally expensive.

Katul *et al.* (2004) suggested that a logical choice was a 1.5-closure model in which a budget equation for TKE must be explicitly considered, such models, knows as two-equations models or $k - \varepsilon$ models. In studies of the canopy flow, terms source/sink due to the drag caused by the vegetation have been included in the transport equations of momentum, TKE and the turbulent kinetic dissipation rate (ε) from $k - \varepsilon$ models (Svensson and Häggkvist, 1990; Katul *et al.*, (2004)). The $k - \varepsilon$ models uses the eddy-viscosity concept to model the Reynolds stress. This concept describes the turbulence as a diffusion process, with the aid of a local isotropic turbulent viscosity. The eddy-viscosity concept gives the isotropic turbulent intensities, i.e., $u_j^2 = (2/3)k$, $i = 1, 2, 3$. Furthermore, the turbulent fluxes $-\overline{u_i'\phi'}$ for a quantify ϕ is assumed to have the same direction as the mean-gradient vector of ϕ. Thus, the eddy-viscosity concept cannot distinguish between anisotropy and inhomogeneity of the turbulence. Thus, an inherent advantage of using a RSM model is that it unhooks the Reynolds stress from the mean velocity field as SKE model.

The aim of the present study is to examine the feasibility of the RMS of providing vertical profiles of the mean velocity, turbulence intensity and Reynolds stress in a roughness sub-layer occupied by forest canopy. In the present work the comparisons of these flow statistics obtained from RSM with predictions from SKE and experimental data from wind tunnel have been made.

2. The Governing Equations

For steady-state and incompressible flow, in the neutral-stability condition, the continuity and momentum equations can be written as follows:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \rho u_i u_i}{\partial x_j} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_i}{\partial x_i}\right)\right] + \frac{\partial}{\partial x_j}(-\rho\overline{u_i'u_j'}) + S_i \tag{2}$$

where ρ is the fluid density, p is the pressure, μ is the dynamic viscosity, δ_{ij} is the Kronecker delta, $\overline{u_j'u_i'}$ is the Reynolds stress tensor and S_i is the source term. The turbulent viscosity, μ_t, in both RSM and SKE turbulence models is evaluated from the TKE and its dissipation rate:

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \tag{3}$$

where $C_\mu = 0.09$ is an empirical constant.

2.1. Reynolds Stress Model

In the Reynolds Stress model (Launder *et al.*, 1975; Gibson and Launder, 1978; Launder, 1989), the transport equations are solved for the individual Reynolds stresses, $\overline{u_i'u_j'}$. The individual Reynolds stresses are then used to obtain the closure for the momentum equation, Eq. (2). The transport equation in the Reynolds Stress model is defined as:

$$\begin{aligned}\frac{\partial}{\partial x_k}(\rho\overline{u_i'u_j'}) \quad &= \quad -\frac{\partial}{\partial x_k}\left[\rho\overline{u_i'u_j'u_k'}\right] + p(\overline{\delta_{kl}u_i' + \delta_{ik}u_j'}) + \frac{\partial}{\partial x_k}\left[\mu\frac{\partial}{\partial x_k}\left(\overline{u_i'u_j'}\right)\right] - \\ &\quad -\rho\left(\overline{u_i'u_k'}\frac{\partial u_j}{\partial_k} + \overline{u_j'u_k'}\frac{\partial u_i}{\partial x_k}\right) + p\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right) - 2\mu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}\end{aligned} \tag{4}$$

2.2. Standard $k - \varepsilon$ Model

In the framework of the SKE turbulence model (Launder and Spalding, 1972), the turbulence kinetic energy, k, and its dissipation rate, ε, are obtained from the following transport equations:

$$\frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_i}\left[(\mu + \frac{\mu_t}{\sigma_k})\frac{\partial k}{\partial x_j}\right] + G_k - \rho\varepsilon + S_k \tag{5}$$

$$\frac{\partial}{\partial x_i}(\rho\varepsilon u_i) = \frac{\partial}{\partial x_i}\left[(\mu + \frac{\mu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial x_j}\right] + C_{1\varepsilon}\frac{\varepsilon}{k}G_k - C_{2\varepsilon}\rho\frac{\varepsilon^2}{k} + \varepsilon \tag{6}$$

In Eqs. (5) and (6), G_k represents the generation of the TKE due to the mean velocity gradients, σ_k and σ_ε are the turbulent Prandtl numbers for k and ε, respectively. The model constants are $C_{1\varepsilon} = 1.44$ and $C_{2\varepsilon} = 1.92$. In the SKE model the Reynolds stress are modelled by using an effective turbulent viscosity, μ_t, such that $-\overline{u'w'}$ is given by

$$\overline{u'w'} = \mu_t \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \tag{7}$$

2.3. Source terms for the momentum, k and ε equations

The Eqs. (2), (4) and (5) include additional terms source/sink for drag due to canopy elements. The model for S_u depends on the form drag coefficient for the canopy:

$$S_u = -\frac{1}{2}\rho A(z) C_D |U| U_i \tag{8}$$

The term for S_k arises because vegetation elements break the mean flow motion and generate the wake turbulence ($\approx A(z) C_D |U|^3$). In the present study the S_k is given by (Svensson and Häggkvist, 1990):

$$S_k = -\frac{1}{2}\rho A(z) C_D |U|^3 \tag{9}$$

The S_ε model is a logical extension of Kolmogorov´s relation (Liu *et al.*, 1996), which in the present work is modelled by (Svensson and Häggkvist, 1990):

$$S_\varepsilon = -\frac{1}{2}\rho C_{4\varepsilon} A(z) C_D |U|^3 \tag{10}$$

where $C_{4\varepsilon} = 1.95$ is an empirical constant.

3. Wind Tunnel Studies

The experiments in the neutral-stratified conditions were performed an open-return blow-through tunnel. The working section of the tunnel are 25 m long, 2.4 m wide and 1.5 in tall. The wind tunnel experiments were fully detailed in Chen *et al.*, (1995). The three components of the wind vector (u, v, w) were made using a tri-axial fibre-film hot-wire probe (Dantec Measurement Technology, Skovlunde, Denmark,model 55r91) connected to a constant temperature anemometer system (Dantec Model 56C01).

The turbulent flow corresponding to the atmospheric boundary layer was generated in the tunnel by a combination of vertical spires, transverse boards, and large and small roughness elements placed successively downwind from the inlet, in a total length of about 7 m. Immediately downwind the small roughness elements there is a 6 m long reduced scale model of a forest (Novak *et al.*, 2000; Novak *et al.*, 2001). In the present study tree density of 125 trees m^{-1} was numerically simulated equivalent to a total leaf-area index of 1.7. In wind tunnel experiments the turbulence statistics were measured at about $x = 4.3$, 5.1 and 5.9 m. At each of these locations, experimental vertical profiles were measured at four positions around a tree and all data presented were averages of these with equal weighting.

4. Numerical Methods and Boundary Conditions

The 2-D turbulent flow within and above the forest canopy was modelled at wind tunnel scale. The forest model was 6 m in length and 0.15 m in height and the domain had a total of 19 m in length and 1.5 m in height and represented a vertical plane at the center of the 3-D geometry of the wind tunnel, as shows Fig. (1). The boundary layer development section, which precedes the forest model, was 7 m long. The size and distribution of the transverse boards, and the large and small roughness elements were the same (two-dimensional equivalent) as the wind-tunnel geometry.

The simulation of the turbulent flow was performed by solving the previously mentioned transport equations for the conservation of mass, momentum, k and ε, using a commercial CFD solver, FLUENT 6.12.16. The SIMPLE algorithm of Patankar, (1980) was used in the continuity, TKE and their disspation rate.The upwind boundary was defined as a velocity inlet, with a logarithmic expression of the velocity given by (Richards and Hoxey, 1993),

$$u(z) = \frac{u_*}{k_v} ln \frac{z + z_o}{z_o} \tag{11}$$

where $u(z)$ is the mean wind speed, $k_v = 0.4$ is the von Karman's constant, $u_* = 0.82$ is the friction velocity, $z_o = 0.0148m$ is the roughness length and z is the height above the ground. The TKE and dissipation rate were assigned profiles based on the value of u_*, using relationships recommended by Richards and Hoxey, (1993),

$$k = \frac{u_*^2}{\sqrt{C_\mu}} \tag{12}$$

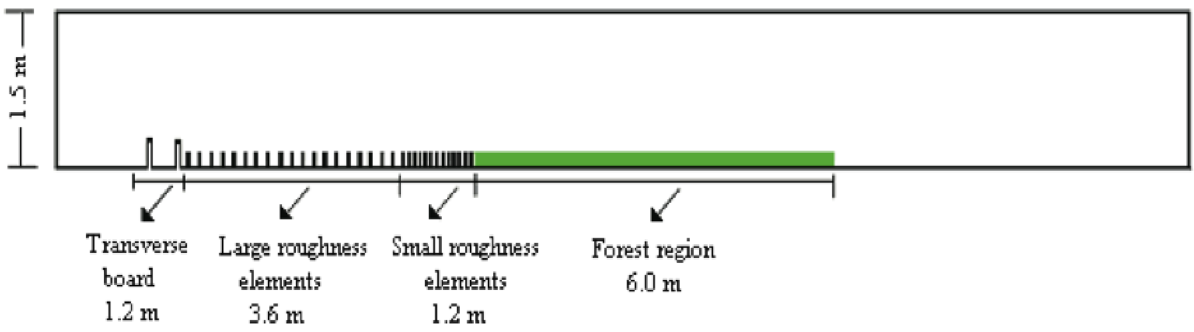

Figure 1: Schematic representation of the 2-D Domain.

and

$$\varepsilon = \frac{u_*^3}{k_v(z+z_o)} \tag{13}$$

The bottom of the wind tunnel was defined as a no-slip wall. In addition, the wall boundary conditions for the $k-\epsilon$ model was treated with the classical log wall function. The top of the modelled domain was defined as a symmetry boundary and the downwind boundary was defined as an outflow boundary.

In the present work, it was used the experimental vertical profiles for the drag coefficient, C_d, calculated according to:

$$\frac{d\overline{u'w'}}{dz} = -\frac{1}{2}C_d(z)A_f(z)\bar{u}^2(z) \tag{14}$$

where $A_f(z)$ is the frontal leaf-area distribution.

5. Results and Discussion

Figure (2) shows the comparison between the measured and modelled SKE and RSM for the vertical velocity profiles. Both SKE and RSM models results agree well with the measurements. There are generic features in all these data sets that the models reproduce well:

(I) A decay exponentially of the wind velocity from the tree top ($z/h_f < 1$), leading to low velocities inside the canopy;

(II) The typical logarithmic profile of the wind speed at heights greater than h_f

(III) An inflection point in the mean velocity profile at the tree top.

Figure (3) shows the wind profiles within-canopy measured and predicted with SKE and RSM turbulence models. Approximately above of 0.03 m two distinct regions of air flow can be observed: (a) an upper region (0.11 m < z < h) where occur a great wind shear and (b) a middle region (0.03 m < z < 0.11 m) where wind shear was relatively weak. It is important to note that numerical results from SKE were not able to capture well this behavior.

Figure (4) shows the comparison between predicted turbulence intensity profiles from SKE, RSM with measurements. In all numerical simulations, turbulence intensity showed discrepancies with the measurements. However, the numerical results were able to capture the maximum turbulence intensity near the top of the canopy, in agreement with the measured values from the wind tunnel experiments.

Figure (5) presented a comparison of the Reynolds tensor profiles obtained from SKE and RSM turbulence models against the measurements. Clearly, the RSM model reproduce well the measured Reynolds tensor than that SKE model. A logical question that was whether this poor performance of the SKE model was connected with the formulation of the Reynolds stress in this model. The measured and modelled components of the Reynolds tensor reveal that below $z/h_f = 0.50$, the magnitudes of the various components were small compared to their values near the top of the canopy. In addition, the computed Reynolds stress with both models decrease to zero near the ground. These results of the Reynolds stress in the lower canopy were consistent with results from Wilson and Shaw (1977), who computed using a higher-order closure model.

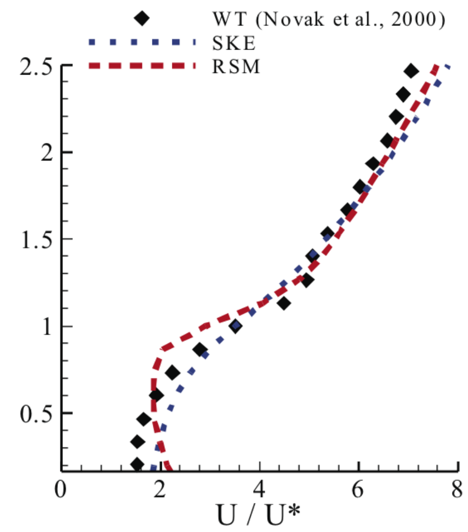

Figure 2: Comparisons between measured and modelled vertical velocity profiles by SKE and RSM turbulence models.

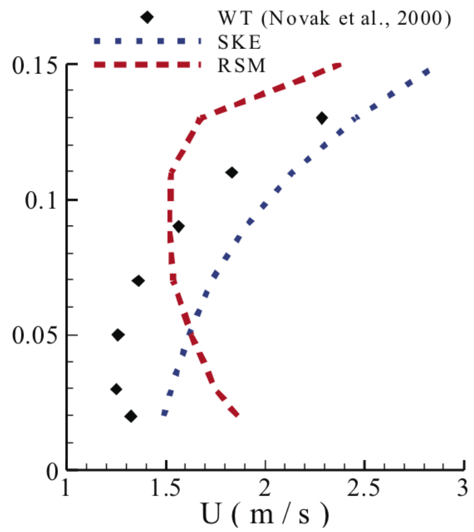

Figure 3: Wind speed profile within the forest canopy.

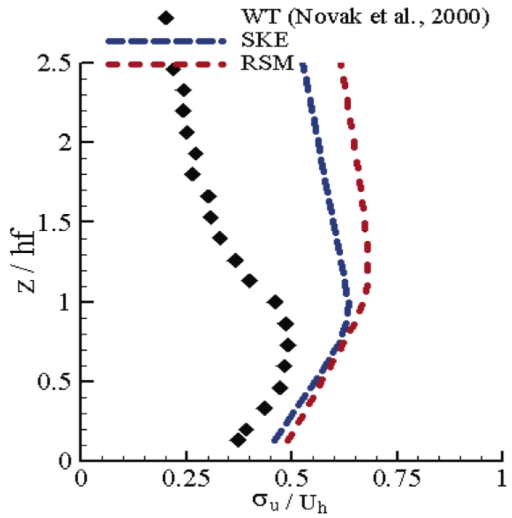

Figure 4: Comparisons between measured and modelled turbulence intensity profiles by SKE and RSM turbulence models.

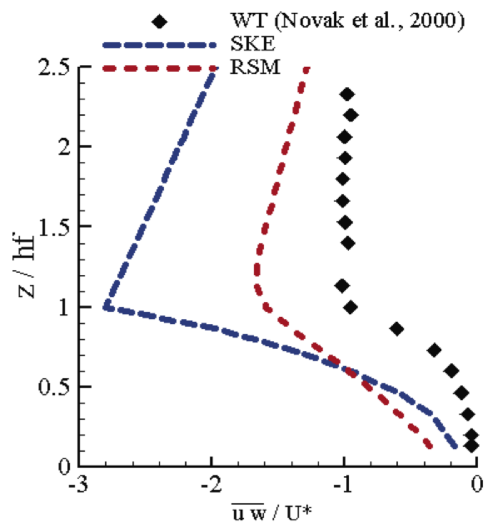

Figure 5: Comparisons between measured and modelled Reynolds stress velocity profiles by SKE and RSM turbulence models.

6. Conclusion

The RSM and SKE turbulence models were used to simulate turbulent air flow within and above a forest canopy. The interaction between the atmosphere and forest was represented by sink term in the transport equation of momentum, for both RSM and SKE models and source terms in the transport equations of k and ε only in the SKE model. The modelled wind speed in both models showed good agreed with wind tunnel measurements. The wind profile was characterized by a great wind shear at the canopy-atmosphere interface and by a region of relatively weak wind shear in the middle canopy.

The predicted turbulence intensity with both turbulence models, within and above the canopy, showed small discrepancies with the measurements. The magnitudes of the modelled Reynolds tensor below of $z/h_f = 0.50$ were small compared to their values near the top of the canopy. Clearly, none of the two models reproduce well the measured Reynolds tensor above the canopy though the SKE was much worse than the RSM. There is no clear advantage in using the RSM versus the SKE in the simulation of the canopy flow, because the SKE model was computationally three times faster than the RSM model.

7. Acknowledgements

The authors acknowledge support from CNPq. The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of this institution.

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